national accelerator laboratory

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0402

TUNE SHIFTS DUE TO THE ORBIT SAGITTA IN BENDING MAGNETS

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I.

The arrangement of quadrupoles in the main ring is symmetric in the horizontal and vertical directions so that the tunes of betatron oscillations, ν_x and ν_y, are identical when all quadrupoles have the same field gradient. Since the beam enters into and exists from each bending magnet slightly off the normal direction, there is a small amount of focusing action in the vertical direction at both ends of the magnet. The corresponding defocusing action in the horizontal direction is cancelled by the equal amount of focusing action within the bending magnet and there is no net effect. Consequently, the vertical tune is slightly above the horizontal tune,

$$(\nu_x - \nu_y)_{\text{edge rotation}} = -0.038.$$

This difference is independent of the energy of the beam.

For some time until quite recently, it had been somewhat of a mystery that the measured value of ν_x is <u>above</u> that of ν_y, approximately 0.08. When the closed orbit of the beam was carefully centered, it was found out that the difference is energy dependent. The present situation seems to be:

$$\nu_x - \nu_y = -0.08 \sim -0.10 \text{ at injection,}$$

$$= 0 \text{ sometime before the transition (17.5 GeV),}$$

$$= +0.08 \text{ at high energies.}$$

Since the gradient length varies slightly from quadrupole to quadrupole, it is entirely possible that the total focusing action of all quadrupoles is different in two directions. Indeed there are approximately 50 so-called "old" quadrupoles, almost all of them defocusing in the horizontal direction. Field gradients of old quadrupoles are $\sim$1% lower in average compared to field gradients of new or normal quadrupoles so that one expects a positive contribution to the quantity $(\nu_x - \nu_y)$. However, this again is independent of energy.

The purpose of this note is to point out that the orbit sagitta in bending magnets can explain the variation of the tune difference when sextupole fields in bending magnets are properly taken into account. Because of the orbit sagitta, each bending magnet acts as a misaligned sextupole. The sagitta itself is independent of energy but the sextupole field is energy dependent. Sextupole field is composed of three parts, remanent field, eddy current field and the "structural" field due to geometry.

II.

In the lowest-order estimate of sagitta effects, the following assumptions are valid.

1. All dipoles are aligned ideally so that the central orbit is away from the magnetic axis of bending magnets by

$$-\Delta \equiv -(\rho\theta^2_B)/16 \text{ at both ends,}$$

$$+\Delta \equiv (\rho\theta^2_B)/16 \text{ at the center.}$$

Here the radius of curvature ρ is 747.813 m and the bending angle θ_B is 8.1178 mrad so that $\Delta = 3.1$ mm.

2. Sextupole field in a bending magnet is

$$B_x = B''Xy,$$

$$B_y = (B''/2)(X^2 - y^2).$$

The horizontal and vertical coordinates measured from the magnetic axis are X and y, respectively. All B1 magnets and B2 magnets have the same sextupole strength,

$$B'' = B''_1 \text{ in all B1's and } B''_2 \text{ in all B2's.}$$

3. The distortion of the closed orbit due to sextupole field is negligible.

The horizontal coordinate, x, of the free oscillation is related to the coordinate X by the relation

$$X = x + X_p\ \delta_p + w,$$

$$w(s) = -s^2/(2\rho) + (\theta_B s)2 - \Delta\ ;\ 0 \le s \le L.$$

Here X_p is the dispersion parameter of the closed orbit, δ_p is the fractional momentum deviation and L is the length of the bending magnet (6.07 m). Equations of motion for free oscillations are:

horizontal (y=0)

$$x'' + x(B'/B\rho) = -B''(x+X_p\ \delta_p + w)^2/(2B\rho)$$

vertical (x=0)

$$y'' - y(B'/B\rho) = B''(X_p\ \delta_p + w)y/(B\rho).$$

In general, B" is a function of <u>s</u> since the sextupole field may be different at different places in the magnet. However, for the sake of simplicity, B" here is regarded as an "average" value. The meaning of terms on the right hand side of these equations is obvious:

$B''x^2$: regular sextupole driving term for $3\nu_x$ = integer resonances,

$B''X_p\ \delta_p x$ and $B''X_p\ \delta_p y$: momentum dependence of tunes,

$B''(X_p\ \delta_p + w)^2$: dipole term whose effect is neglected here,

$B''wx$ and $B''wy$: tune shifts due to sagitta.

In the lowest order, tune shifts are

$$(\Delta\nu_x) = (1/4\pi\ B\rho)B_1'' \sum_{\text{All B1}} \int_0^L ds\ \beta_x(s) \langle X_p(s)\delta_p + w(s)\rangle$$

$$+(\text{contribution from all B2}),$$

$$(\Delta\nu_y) = -(1/4\pi\ B\rho)B_1'' \sum_{\text{All B1}} \int_0^L ds\ \beta_y(s) \langle X_p(s)\delta_p + w(s)\rangle$$

$$+(\text{contribution from all B2}).$$

Geometrical quantities are easy to evaluate from the SYNCH standard output:

$$\Sigma \int_0^L \beta_x(s)\ X_p(s)ds = 4.981 \times 10^5\ m^3 \text{ for B1,}$$
$$= 2.315 \times 10^5\ m^3 \text{ for B2,}$$

$$\Sigma \int_0^L \beta_x(s)\ w(s)ds = 169.7\ m^3 \text{ for B1,}$$
$$= 96.54\ m^3 \text{ for B2,}$$

$$\Sigma \int_0^L \beta_y(s)\ X_p(s)ds = 2.632 \times 10^5\ m^3 \text{ for B1,}$$
$$= 4.133 \times 10^5\ m^3 \text{ for B2,}$$

$$\Sigma \int_0^L \beta_y(s)\ w(s)ds = 91.57\ m^3 \text{ for B1,}$$
$$= 176.9\ m^3 \text{ for B2.}$$

From these

$$(\Delta\nu_x) = \langle 0.396(B_1''/B\rho) + 0.184(B_2''/B\rho)\rangle \times 10^5\ \delta_p$$
$$+\langle 13.5(B_1''/B\rho) + 7.68(B_2''/B\rho)\rangle.$$

$$(\Delta\nu_y) = -\langle 0.209(B_1''/B\rho) + 0.329(B_2''/B\rho)\rangle \times 10^5\ \delta_p$$
$$-\langle 7.29(B_1''/B\rho) + 14.1(B_2''/B\rho)\rangle.$$

III.

A simple analytic formula gives the eddy current sextupole field,

$$B'' \ (\mathrm{kG/m^2}) = 0.111 \ dB/dt \quad (\mathrm{kG/sec}) \text{ in B1,}$$
$$= 0.0832 \ dB/dt \quad (\mathrm{kG/sec}) \text{ in B2.}$$

The magnetic field B is parabolic from t=0 (injection) to t=0.2 sec and linear after that. For 100 GeV/c/sec acceleration (after t=0.2 sec),

$$B'' \text{ (eddy current)} = 2.476 \ t \text{ for B1,}$$
$$= 1.856 \ t \text{ for B2;} \quad t < 0.2 \text{ sec,}$$
$$= 0.495 \text{ for B1,}$$
$$= 0.371 \text{ for B2;} \quad t < 0.2 \text{ sec.}$$

The remanent sextupole field and the "structural" sextupole field can be obtained from the momentum dependence of tunes at two different energies. Data used here are[1]

$$\Delta\nu_x = (-23.4 - 206)\delta_p$$
$$\Delta\nu_y = (-23.4 + 169)\delta_p \text{ at injection (cp=8.889 GeV);}$$
$$\Delta\nu_x = (-23.4 + 2.8)\delta_p,$$
$$\Delta\nu_y = (-23.4 - 2.2)\delta_p \text{ at 150 GeV.}$$

The familiar term, $-23.4\delta_p$, is due to the chromatic aberration of quadrupoles. From these and eddy current contributions given above, one finds

$$B'' \text{ (remanent)} = -1.244 \text{ (kG/m}^2\text{) for B1,}$$

$$= -0.810 \text{ (kG/m}^2\text{) for B2;}$$

$$B'' \text{ (structural)} = 0.2043 \times 10^{-3} \text{ (B}\rho\text{) for B1,}$$

$$= 0.1206 \times 10^{-3} \text{ (B}\rho\text{) for B2.}$$

Tune shifts due to the edge rotation of bending magnets have already been mentioned in I:

$$\Delta\nu_x = 0$$

$$\Delta\nu_y = 0.038.$$

In addition to this, variation of the gradient length from quadrupole to quadrupole can contribute to $(\nu_x - \nu_y)$. Data for quadrupoles presently used in the main ring are unfortunately not available but those for quadrupoles in the ring in April, 1972 should be adequate for the present purpose. (Since that time, only seven 7-foot and three 4-foot quadrupoles have been replaced.) Also, data on 4-foot quadrupoles are not as reliable as those on 7-foot ones so that the variations in all 4-foot quadrupoles are ignored here. The tune difference is $\nu_x - \nu_y = 0.084$.

Splitting this difference equally, one gets

$$\Delta\nu_x = 0.042,$$

$$\Delta\nu_y = -0.042.$$

Total tune shifts are shown in Figure 1 as a function of the momentum. The difference, $\nu_x - \nu_y$, is shown in Figure 2. The agreement with measurements is good except at high energies. The measured value, $\nu_x - \nu_y \cong 0.08$, is somewhat higher than 0.05 given in Figure 2.

REFERENCES

1. Almost all tune measurements have been done by R. Stiening and D. Edwards. Unfortunately, results are not available even as an informal report. However, the momentum dependence of tunes at 75 GeV and at 150 GeV are discussed in Accelerator Experiment 23, September 12, 1972.

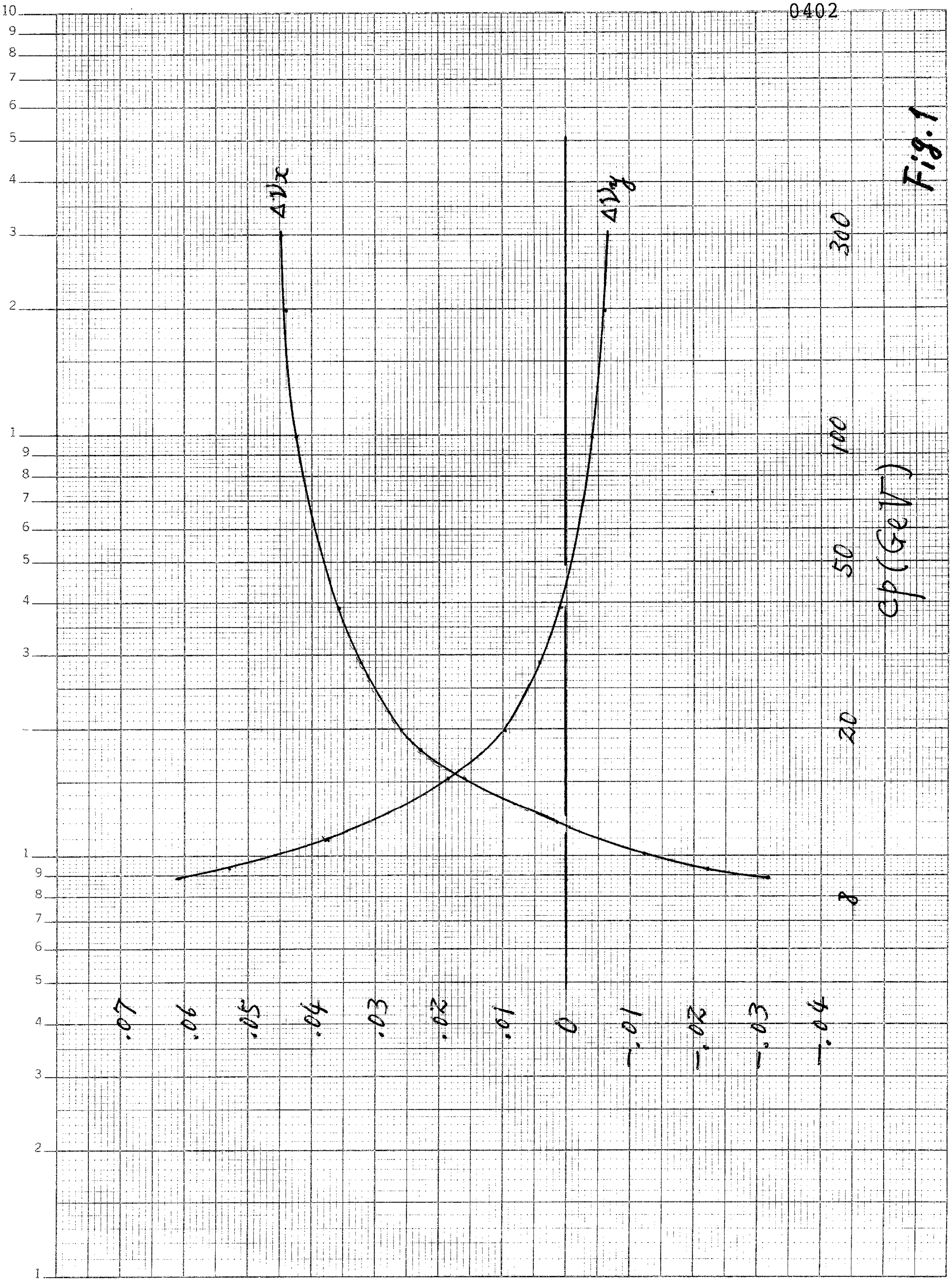
Fig. 1
Cp (GeV)
ΔVx
ΔVy
300
100
50
20
8
.07
.06
.05
.04
.03
.02
.01
0
-.01
-.02
-.03
-.04

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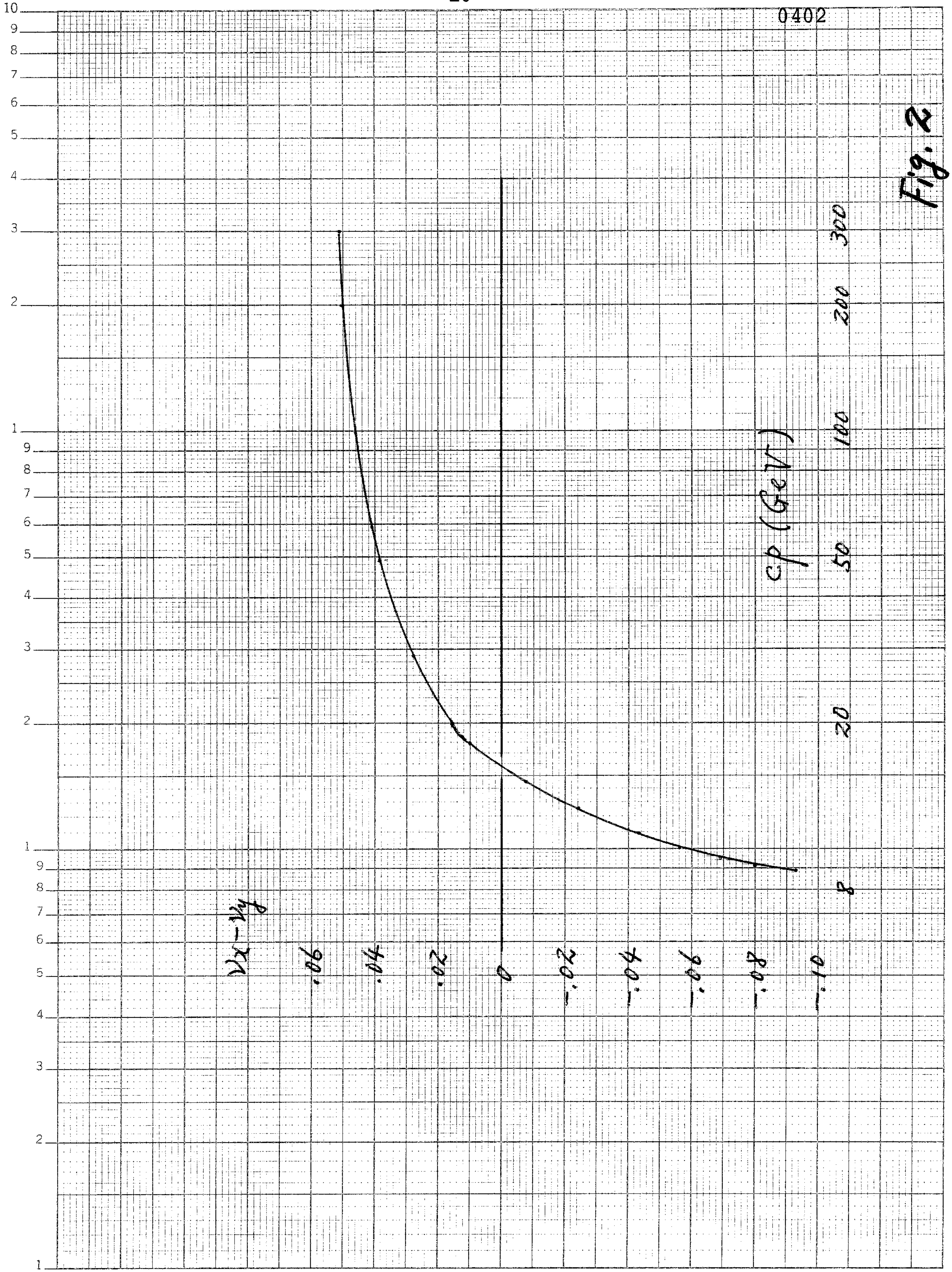

Fig. 2